CLASS - XI

Chapter – 9

SEQUENCES AND SERIES

MODULE – 1 of 3

Distance Learning Programme: An initiative by AEES, Mumbai

CONTENTS

1. Revised Curriculum

- **DELETED PORTION (FOR THE SESSION 2020-2021)**
  Formulae for the following special sums - Σk, $\Sigma \mathrm{k}^2$, Σk^3.

➢ REVISED PORTION (FOR THE SESSION 2020-2021)

COURSE STRUCTURE **Total Periods–168 [35 Minutes Each]**

<table>
<tr><th>No.</th><th>Units</th><th>No. of Periods</th><th>Marks</th></tr>
<tr><td>I</td><td>Sets and Functions</td><td>43</td><td>23</td></tr>
<tr><td>II</td><td>Algebra</td><td>41</td><td rowspan="3">30</td></tr>
<tr><td rowspan="2"></td><td>(a) Complex Numbers and Quadratic Equations, (b) Linear Inequalities, (c) Permutations and Combinations</td><td></td></tr>
<tr><td>(d) Sequence and Series –
Sequence and Series, Arithmetic Progression (A. P.), Arithmetic Mean (A.M.), Geometric Progression (G.P.), General term of a G.P., Sum of n terms of a G.P., Infinite G.P. and its sum, Geometric mean (G.M.), Relation between A.M. and G.M.</td><td>(08 periods of 35 Minutes each)</td></tr>
<tr><td>III</td><td>Coordinate Geometry</td><td>33</td><td>10</td></tr>
<tr><td>IV</td><td>Calculus</td><td>30</td><td>7</td></tr>
<tr><td>V</td><td>Statistics and Probability</td><td>21</td><td>10</td></tr>
<tr><td colspan="2">Total</td><td>168</td><td>80</td></tr>
<tr><td colspan="2">Internal Assessment</td><td></td><td>20</td></tr>
</table>

2. Sequence

A sequence is an enumerated collection of objects in which repetitions are allowed and order matters

- **For example**
  (M, A, R, Y) is a sequence of letters with the letter 'M' first and 'Y' last.
  This sequence differs from (A, R, M, Y).

  Also, the sequence (1, 1, 2, 3, 5, 8), which contains the number 1 at two different positions, is a valid sequence.

Rank (Index)

The position of an element in a sequence is its *rank* or *index*.

It is the natural number for which the element is the image.

Terms

The various numbers occurring in a sequence are called its terms.
We denote the terms of a sequence by a_1, a_2, a_3, ..., a_n, …, etc., the subscripts (1, 2, 3, …., n, ….) denote the position of the term.

*The nth term is called the **general term** of the sequence.*

Finite & Infinite Sequences

A sequence containing finite number of terms is called a **finite** sequence.

Some Examples ↓

Example-1

Sequence of natural numbers less than and equal to 100

i.e. 1, 2, 3, 4, 5, 6, …….., 98, 99, 100.

Example-2

Sequence of First ten odd numbers

i.e. 1, 3, 5, 7, 9.

A sequence is called **infinite**, if it is not a finite sequence.

Some Examples ↓

Example-1

The sequence of even numbers

i.e. 2, 4, 6, 8, 10, 12, ……………… ∞ terms

Example-2

The sequence of perfect square natural numbers

i.e. 1, 4, 9, 16, 25, 36, ……………… ∞ terms

3. Progressions

The Sequences following specific patterns are called *progressions*.

Example of some progressions

- 1, 3, 5, 7, 9, 11.
- 1, 3, 9, 27, 81, 243, …..
- 25, 20, 15, 10, 5, 0, -5.
- 1, 4, 9, 16, 25, 36, 49.

In some cases, an arrangement of numbers has no visible pattern
Such as 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ………
But this sequence is generated by the recurrence relation given by
$a_1 = a_2 = 1$
$a_n = a_{n-2} + a_{n-1}, n > 2$
This sequence is called ***Fibonacci sequence***.

Fibonacci sequence – 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ………

Mathematics Progressions

- ***Arithmetic progression*** **(A.P)**

  Sequence of numbers such that the difference of any two successive members of the sequence is a constant

  Example – 1, 4, 7, 10, 13, …… (difference is 3)

- ***Geometric progression*** **(G.P)**

  Sequence of numbers such that the quotient of any two successive members of the sequence is a constant

  Example – 1, 4, 16, 64, 256, …… (quotient is 4)

- ***Harmonic progression*** **(H.P)**

  Sequence of numbers such that their reciprocals form an arithmetic progression

  Example – 1/1, 1/4, 1/7, 1/10, 1/13, …… (reciprocals are in A.P.)

4. Series

Let a_1, a_2, a_3,…,a_n, be a given sequence, then,

The expression $a_1 + a_2 + a_3 +,\ldots + a_n + \ldots$ is called the *series associated with the given sequence* .

In compact form ↓

Series is $\sum_{k=1}^{n} a_k$. $(Here\ the\ notation\ \Sigma\ is\ a\ Greek\ letter\ called\ sigma)$

Some Examples

Example 1

Write the first three terms in sequences defined by $a_n = 3n + 1$. Also write the series.

Solution

Here $a_n = 3n + 1$

Substituting $n = 1, 2, 3$, we get

$a_1 = 3(1) + 1 = 4, \quad a_2 = 3(2) + 1 = 7, \quad a_3 = 3(3) + 1 = 10$

Sequence → 4, 7, 10

Series → 4 + 7 + 10

Or Series → $\boldsymbol{\sum_{n=1}^{3} a_n \quad = \sum_{n=1}^{3}(3n+1) = 4 + 7 + 10}$

Example 2

What is the 15th term of the sequence defined by $a_n = (n + 1)(2 - n)$?

Solution

Here, $a_n = (n + 1)(2 - n)$

For 15^{th} term, put n = 15

$a_{15} = (15 + 1)(2 - 15) = 16 \times -13 = -208$

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